SEM-2 (MATHEMATICS GENERAL)

ANUP RAY

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Rule 5: If the equation $M\,dx + N\,dy = 0$ is of the form $f_1(xy)\,y\,dx + f_2(xy)\,x\,dy = 0$, then $\frac{1}{Mx - Ny}$ is an integrating factor of $M\,dx + N\,dy = 0$, provided $(Mx - Ny) \neq 0$

Ex: Solve $(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$

Ans: Compairing the given equation with the equation $M\,dx + N\,dy = 0$, we get

$M = (xy\sin xy + \cos xy)\,y$ and $N = x(xy\sin xy - \cos xy)$

Given equation is of the form $f_1(xy)\,y\,dx + f_2(xy)\,x\,dy = 0$.

Again $Mx - Ny$

$= xy\,[xy\sin xy + \cos xy - xy\sin xy + \cos xy]$

$= 2xy\cos xy \neq 0$

Hence $I.F. = \frac{1}{2xy\cos xy}$

Multiplying the given equation by I.F. we get,

$$\frac{1}{2}\left(y\tan xy + \frac{1}{x}\right)dx + \frac{1}{2}\left(x\tan xy - \frac{1}{y}\right)dy = 0$$

which must be exact and so by the usual rule, the solution of the given equation is

$$\frac{1}{2}\int\left(y\tan xy + \frac{1}{x}\right)dx\ [\text{Taking } y \text{ as constant}] + \frac{1}{2}\int -\frac{1}{y}\,dy\ [\text{Terms free from } x] = 0$$

or $\frac{1}{2}[\log\sec xy + \log x] - \frac{1}{2}\log y = \frac{1}{2}\log c$

or $\frac{x}{y}\sec xy = c$

Home work:

1) Solve:

i) $y(1+xy)\,dx + x(1-xy)\,dy = 0$

2) $(x^3y^3 + x^2y^2 + xy + 1)\, y\, dx + (x^3y^3 - x^2y^2 - xy + 1)\, x\, dy = 0$

3) $y(x^2y^2 + 2)\, dx + x(2 - 2x^2y^2)\, dy = 0$

4) $(x^2y^2 + xy + 1)\, y\, dx + (x^2y^2 - xy + 1)\, x\, dy = 0$

5) $(x^4y^4 + x^2y^2 + xy)\, y\, dx + (x^4y^4 - x^2y^2 + xy)\, x\, dy = 0$

6) $y(1 - xy)\, dx - x(1 + xy)\, dy = 0$

7) $(xy^2 + 2x^2y^3)\, dx + (x^2y - x^3y^2)\, dy = 0$

Answer of Home work problems:

1) a) yes b) yes

2) $y^3 = 3x^3 \log x + cx^3$

3) $x^2 - y^2 - 2xy = c$

4) $\frac{x^4}{4} - x^3y + xy^2 - \frac{y^4}{4} = 0$

Home work

1) ~~what~~ what do you mean integrating factor. Find three integrating factor of $x\,dy - y\,dx = 0$

2) Find the value of λ for which the differential Equation $(2xe^y + 3y^2)\, dy + (3x^2 + \lambda e^y)\, dy = 0$ is Exact. Hence solve the equation for this value of λ.

3) Solve $(r + \sin\theta - \cos\theta)\, dr + r(\sin\theta + \cos\theta)\, d\theta = 0$